

Solving Equations III

Hints/Guide:

When solving equations that include basic mathematical operations, we must simplify the mathematics first, then solve the equations. For example:

$$\begin{array}{r}
 5(4 - 3) + 7x = 4(9 - 6) \\
 5(1) + 7x = 4(3) \\
 5 + 7x = 12 \\
 -5 \qquad -5 \\
 \hline
 7x = 7 \\
 \frac{7x}{7} = \frac{7}{7} \\
 x = 1
 \end{array}$$

Check: $5(4 - 3) + 7(1) = 4(9 - 6)$
 $5 + 7 = 4(3)$
 $12 = 12$

Exercises: Solve the following equations using the rules listed on the previous pages: **SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.

1. $4x + 8 - 6 = 2(9 - 2)$ 2. $\frac{t}{5} - 7 + 31 = 8(6 - 4)$ 3. $5(t - 4) = 9(7 - 3)$

4. $9 - 5(4 - 3) = -16 + \frac{x}{3}$ 5. $6t - 9 - 3t = 8(7 - 4)$ 6. $7(6 - (-8)) = \frac{t}{-4} + 2$

7. $7(3 - 6) = 6(4 + t)$ 8. $4r + 5r - 6r = 15 + 6$ 9. $3(5 + x) = 5(7 - (-2))$

Equations - Variables on Each Side

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division. Remember the golden rule of equation solving. Examples:

$$\begin{array}{r}
 8x - 6 = 4x + 5 \\
 -4x \quad -4x \\
 \hline
 4x - 6 = 5 \\
 +6 \quad +6 \\
 \hline
 4x = 11 \\
 \frac{4x}{4} = \frac{11}{4} \\
 x = 2\frac{3}{4}
 \end{array}$$

$$\begin{array}{r}
 5 - 6t = 24 + 4t \\
 +6t \quad +6t \\
 \hline
 5 = 24 + 10t \\
 -24 \quad -24 \\
 \hline
 -19 = 10t \\
 \frac{-19}{10} = \frac{10t}{10} \\
 -1\frac{9}{10} = t
 \end{array}$$

Exercises: Solve the following problems:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4r - 7 = 8r + 13$

2. $14 + 3t = 5t - 12$

3. $4x + 5 = 3x - 3$

4. $6y + 5 = 4y - 13$

5. $5x - 8 = 6 - 2x$

6. $7p - 8 = -4p + 6$

Inequalities

Hints/Guide:

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality has many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

Inequality - a statement containing one of the following symbols:

$<$ is less than $>$ is greater than \leq is less than or equal to
 \geq is greater than or equal to \neq is not equal to

Examples:

1. Integers between -4 and 4.



2. All numbers between -4 and 4.



3. The positive numbers.



So, to solve the inequality $-4x < -8$ becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore $x > 2$ is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:



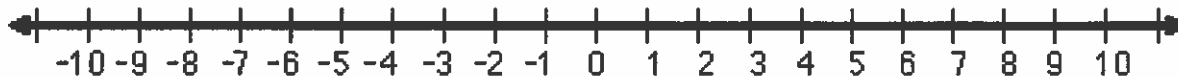
Exercises: Solve the following problems:

No Calculators!

1. $4x > 9$



2. $-5t \geq -15$



3. $\frac{x}{2} \geq 3$



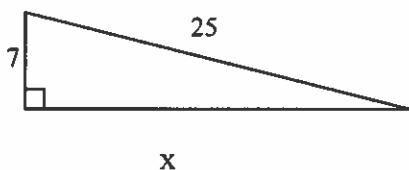
4. $\frac{x}{-4} > 2$



Hints/Guide.

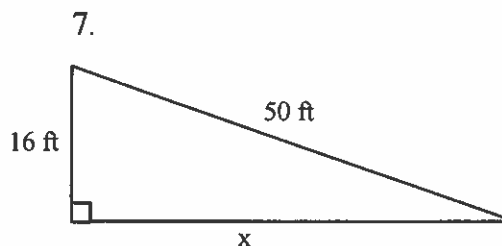
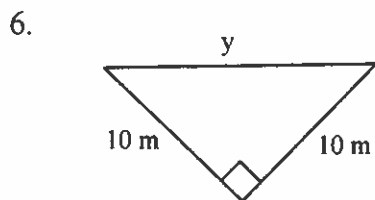
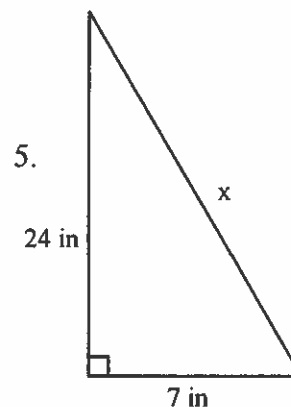
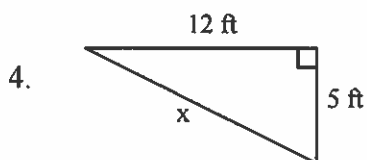
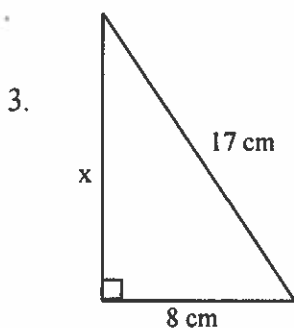
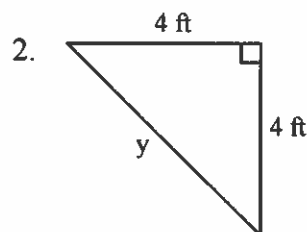
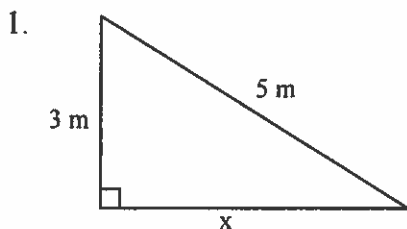
The Pythagorean Theorem states that in a right triangle, and only in a right triangle, the length of the longest side (the side opposite the right angle and called the hypotenuse, or c in the formula) squared is equal to the sum of the squares of the other two sides (the sides that meet to form the right angle called legs, or a and b in the formula). The formula is $a^2 + b^2 = c^2$.

Find the missing side.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 7^2 + x^2 &= 25^2 \\ 49 + x^2 &= 625 \\ -49 &\quad -49 \\ x^2 &= 576 \\ \sqrt{x^2} &= \sqrt{576} \\ x &= 24 \end{aligned}$$

Exercises: Solve for the variable. The above steps should be shown in your work. **SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.



Volume

Hints/Guide:

To find the volume of prisms (a solid figure whose ends are parallel and the same size and shape and whose sides are parallelograms) and cylinders, we multiply the area of the base times the height of the figure. The formulas we need to know are:

The area of a circle is $A = \pi r^2$

The area of a rectangle is $A = bh$

The area of a triangle is $A = \frac{1}{2} b h$

The volume of a prism is

$$V = (\text{Area of Base}) \cdot (\text{Height})$$

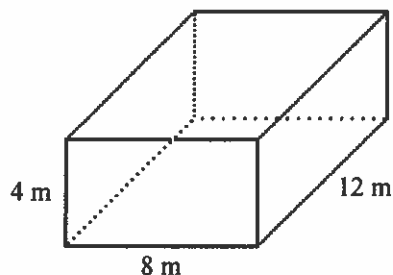
So, the volume of a rectangular prism can be determined if we can find the area of the base and the perpendicular height of the figure.

Exercises: Find the volume of the following figures:

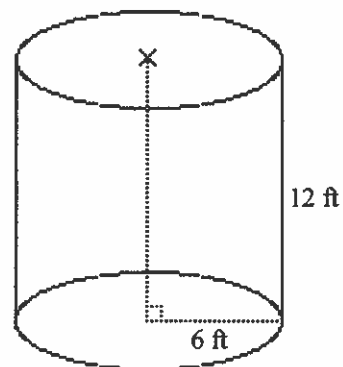
Note: Use $\pi = 3.14$

SHOW ALL WORK: Formula, Substitute, Work, Answer

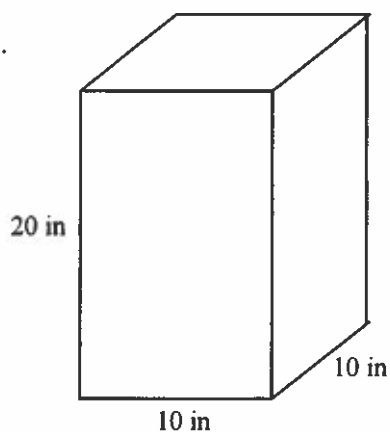
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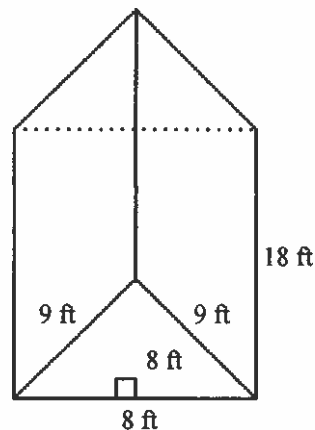
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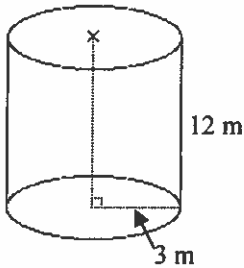
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Surface Area

Hints/Guide:

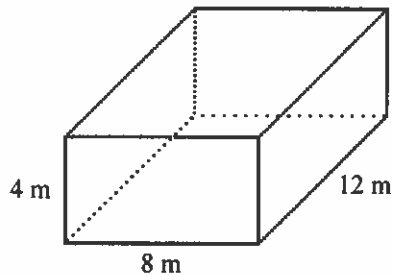
To determine the surface area of an object, we must find the areas of each surface and add them together. For a rectangular prism, we find the area of each rectangle and then add them together. For a cylinder, we find the area of each base and then add the area of the rectangle (the circumference of the circular base times the height) which wraps around to create the sides of the cylinder. For example:



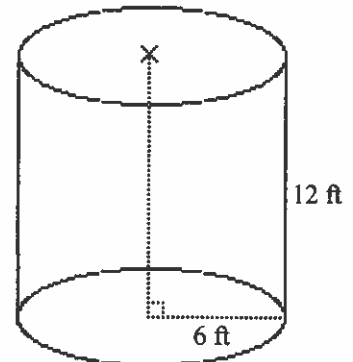
The area of each base is $A = \pi r^2 = 3.14 \cdot 3 \cdot 3 = 28.26 \text{ m}^2$
 and the area of the cylinder "wrap" is
 $A = 2\pi rh$ (which is the circumference of the circle
 times the height of the cylinder)
 $= 2 \cdot 3.14 \cdot 3 \cdot 12$
 $= 226.08$
 So the surface area is $28.26 + 28.26 + 226.08 = 282.6 \text{ m}^2$

Exercises: Determine the surface area of the following figures:
SHOW ALL WORK: Formula, Substitute, Work, Answer

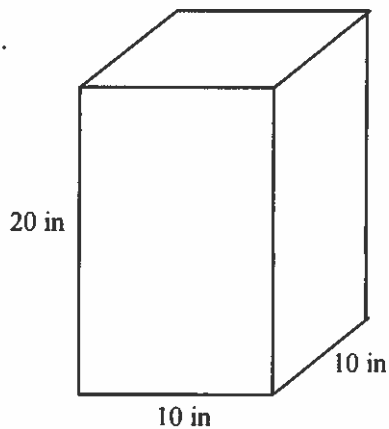
1.



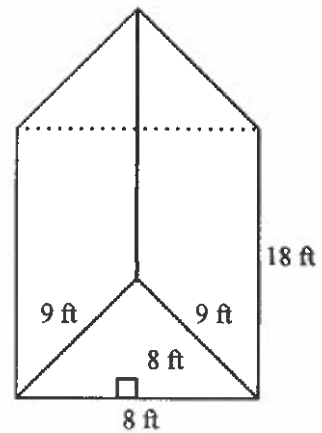
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3.



4.



Problem Solving

Exercises: Solve each problem.

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. The Acme Supply Store sells a security system for \$2150.00 excluding tax. They sold 12 systems. If the total profit on these sales was \$4824.36, how much did each system cost Acme Supply? Show your work.
2. Kristen is paid \$5.60 per hour. She works 6 hours on Saturday, 3 hours on Sunday, and 5 hours on Monday. On Saturday her hourly rate is $1\frac{1}{2}$ times her regular rate and she is paid twice the regular rate on Sunday. How much did she earn in all? Show all work.
3. At the beginning of the week the value of a stock was $32\frac{1}{2}$. On Monday it fell $\frac{1}{2}$, on Tuesday it rose $1\frac{1}{2}$, on Wednesday it rose 3, on Thursday it fell 2, and on Friday it rose $2\frac{1}{2}$. What was the value of the stock at the end of the week? Show all work.
4. Norma is paid \$4.80 per hour. She worked $3\frac{1}{2}$ hours on Friday, 4 hours on Saturday, and $2\frac{1}{2}$ hours on Sunday. On Saturday, her hourly rate was $1\frac{1}{2}$ times her regular pay and on Sunday, it was twice the regular rate. How much did she earn in all? Show all work.
5. The formula which converts Fahrenheit degrees (F) to Celsius degrees (C) is:
$$C = \frac{5}{9} (F - 32).$$
 How many degrees Celsius is 113 F?
6. The formula for the perimeter of a rectangle is $P = 2(l + w)$. If the length is 8 inches and the width is 7 inches, determine the perimeter of the rectangle.

Find Percent of a Number

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut by moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number. For example:

$$4.5\% \text{ of } 240 = 4.5\% \cdot 240 = 0.045 \cdot 240 = 10.8$$

Exercises: Solve for n.

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $305\% \text{ of } 450 = n$

2. $7.5\% \text{ of } 42 = n$

3. $120\% \text{ of } 321 = n$

4. $15\% \text{ of } 54 = n$

5. $0.65\% \text{ of } 320 = n$

6. $800\% \text{ of } 64 = n$

7. $95\% \text{ of } 568 = n$

8. $150\% \text{ of } 38 = n$

9. $215\% \text{ of } 348 = n$

10. $85\% \text{ of } 488 = n$

11. $9.05\% \text{ of } 750 = n$

12. $160\% \text{ of } 42 = n$

13. $60\% \text{ of } 78 = n$

14. $0.4\% \text{ of } 480 = n$

15. $0.10\% \text{ of } 435 = n$

16. $2.4\% \text{ of } 54 = n$